

Operational Semantics Rules

$$\text{[E-SUCC-CTX]} \quad \boxed{\frac{e \rightarrow e'}{\text{succ } e \rightarrow \text{succ } e'}}$$

$$\text{[E-SUCC]} \quad \boxed{\frac{}{\text{succ } i \rightarrow i + 1}}$$

$$\text{[E-PRED-CTX]} \quad \boxed{\frac{e \rightarrow e'}{\text{pred } e \rightarrow \text{pred } e'}}$$

$$\text{[E-PRED]} \quad \boxed{\frac{}{\text{pred } i \rightarrow i - 1}}$$

$$\text{[E-ISZERO-CTX]} \quad \boxed{\frac{e \rightarrow e'}{\text{iszero } e \rightarrow \text{iszero } e'}}$$

$$\text{[E-ISZERO-ZERO]} \quad \boxed{\frac{}{\text{iszero } 0 \rightarrow \text{true}}}$$

$$\text{[E-ISZERO-NONZERO]} \quad \boxed{\frac{i \neq 0}{\text{iszero succ } i \rightarrow \text{false}}}$$

$$\text{[E-IF-CTX]} \quad \boxed{\frac{e_1 \rightarrow e'_1}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \rightarrow \text{if } e'_1 \text{ then } e_2 \text{ else } e_3}}$$

$$\text{[E-IF-TRUE]} \quad \boxed{\frac{}{\text{if true then } e_2 \text{ else } e_3 \rightarrow e_2}}$$

$$\text{[E-IF-FALSE]} \quad \boxed{\frac{}{\text{if false then } e_2 \text{ else } e_3 \rightarrow e_3}}$$

$$\text{[E-CONCAT-CTX1]} \quad \boxed{\frac{e_1 \rightarrow e'_1}{\text{concat } e_1 e_2 \rightarrow \text{concat } e'_1 e_2}}$$

$$\text{[E-CONCAT-CTX2]} \quad \boxed{\frac{e_2 \rightarrow e'_2}{\text{concat } s_1 e_2 \rightarrow \text{concat } s_1 e'_2}}$$

$$\text{[E-CONCAT]} \quad \boxed{\frac{s_3 = s_1 + s_2}{\text{concat } s_1 s_2 \rightarrow s_3}}$$

$$\text{[E-ISEMPTYSTR-CTX]} \quad \boxed{\frac{e \rightarrow e'}{\text{isemptystr } e \rightarrow \text{isemptystr } e'}}$$

[E-ISEMPTYSTR-EMPTY]	$\frac{}{\text{isemptyst}r \text{ ""} \rightarrow \text{true}}$
[E-ISEMPTYSTR-NONEMPTY]	$\frac{s \neq \text{""}}{\text{isemptyst}r s \rightarrow \text{false}}$
[E-STRLEN-CTX]	$\frac{e \rightarrow e'}{\text{strlen } e \rightarrow \text{strlen } e'}$
[E-STRLEN-ZERO]	$\frac{}{\text{strlen } \text{""} \rightarrow 0}$
[E-STRLEN-NONZERO]	$\frac{\text{c is a single char, } s = c + s'}{\text{strlen } s \rightarrow \text{succ strlen } s'}$

Typing Rules

[T-TRUE]	$\frac{}{\text{true} : \text{Bool}}$
[T-FALSE]	$\frac{}{\text{false} : \text{Bool}}$
[T-INT]	$\frac{}{n : \text{Int}}$
[T-STRING]	$\frac{}{s : \text{String}}$
[T-SUCC]	$\frac{e : \text{Int}}{\text{succ } e : \text{Int}}$
[T-PRED]	$\frac{e : \text{Int}}{\text{pred } e : \text{Int}}$
[T-ISZERO]	$\frac{e : \text{Int}}{\text{iszero } e : \text{Bool}}$
[T-IF]	$\frac{e_1 : \text{Bool}, e_2 : T, e_3 : T}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 : T}$
[T-CONCAT]	$\frac{e_1 : \text{String}, e_2 : \text{String}}{\text{concat } e_1 e_2 : \text{String}}$

[T-ISEMPTYSTR]
$$\frac{e : \text{String}}{\text{isemptystr } e : \text{Bool}}$$

[T-STRLEN]
$$\frac{e : \text{String}}{\text{strlen } e : \text{Int}}$$